2.15 Use algebraic manipulation to find the minimum product-of-sums expression for the function $f = (x_1 + x_2 + x_3) \cdot (x_1 + \overline{x_2} + x_3) \cdot (\overline{x_1} + \overline{x_2} + x_3) \cdot (x_1 + x_2 + \overline{x_3})$

Solution:

$$f = (x_1 + x_2 + x_3) \cdot (x_1 + \overline{x_2} + x_3) \cdot (\overline{x_1} + \overline{x_2} + x_3) \cdot (x_1 + x_2 + \overline{x_3})$$

$$= (x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x_3})(x_1 + \overline{x_2} + x_3) \cdot (\overline{x_1} + \overline{x_2} + x_3)$$

$$= (x_1 + x_2)(x_3 + \overline{x_3})(\overline{x_2} + x_3)(x_1 + \overline{x_1})$$

$$= (x_1 + x_2)(\overline{x_2} + x_3)$$

2.20 Design the simplest sum-of-products circuit that implements the function,

$$f(x_1, x_2, x_3) = \sum m(3,4,6,7)$$

Solution:

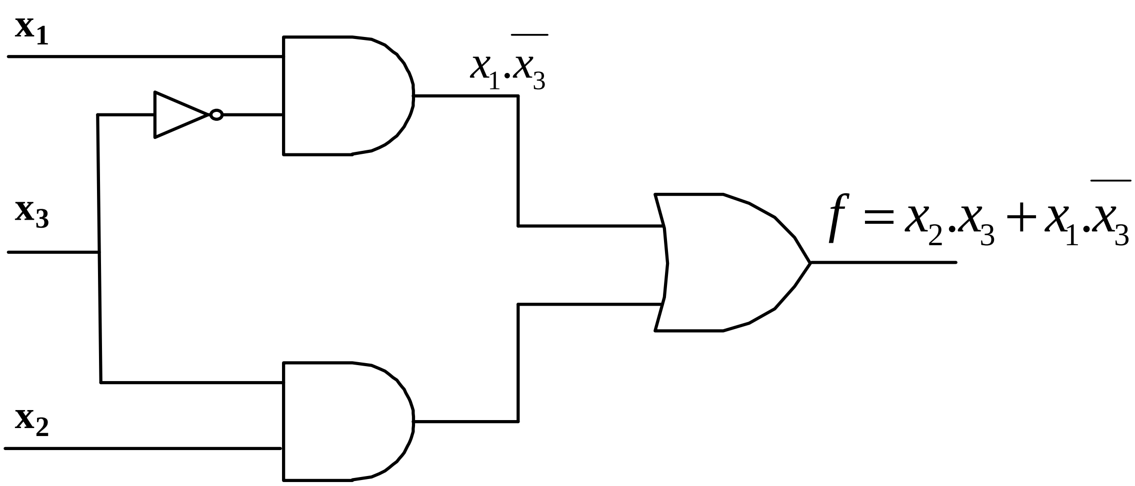
The simplest SOP implementation of the function is

$$f(x_1, x_2, x_3) = \sum m(3,4,6,7) = m_3 + m_4 + m_6 + m_7$$

$$\Rightarrow f = \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.\overline{x_3} + x_1.x_2.\overline{x_3} + x_1.x_2.x_3$$

$$\Rightarrow f = \overline{x_1}.x_2.x_3 + x_1.x_2.x_3 + x_1.\overline{x_2}.\overline{x_3} + x_1.x_2.\overline{x_3}$$

$$\Rightarrow f = x_2.x_3(\overline{x_1} + x_1) + x_1.\overline{x_3}(\overline{x_2} + x_2) = x_2.x_3 + x_1.\overline{x_3}$$



2.23 Design the simplest product-of-sums circuit that implements the function,

$$f(x_1, x_2, x_3) = \prod M(0,1,5,7)$$

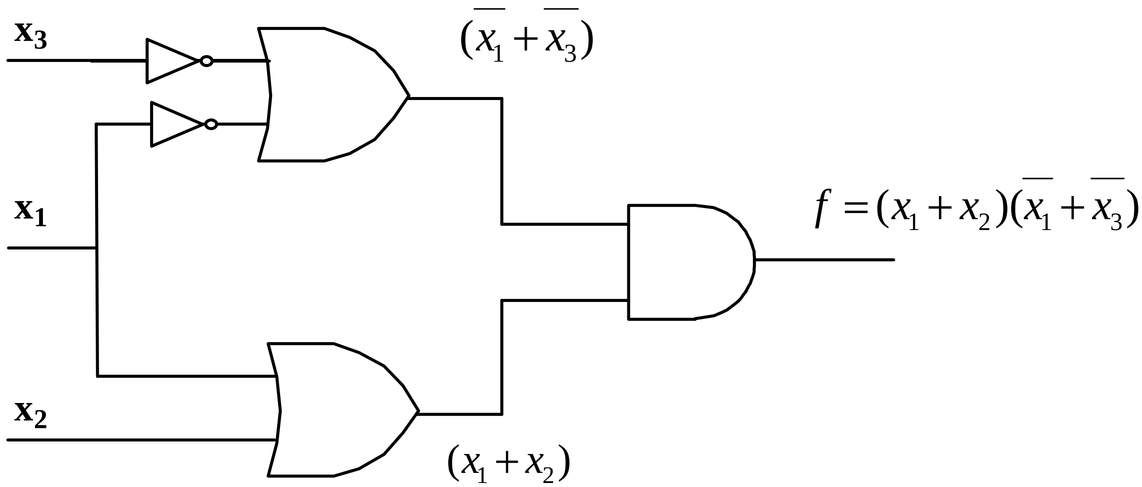
Solution:

$$f(x_1, x_2, x_3) = \prod M(0,1,5,7) = M_0.M_1.M_5.M_7$$

$$f = (x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x_3})(\overline{x_1} + x_2 + \overline{x_3})(\overline{x_1} + \overline{x_2} + \overline{x_3})$$

$$\Rightarrow f = (x_1 + x_2)(x_3 + \overline{x_3})(\overline{x_1} + \overline{x_3})(x_2 + \overline{x_2})$$

$$\Rightarrow f = (x_1 + x_2)(\overline{x_1} + \overline{x_3})$$



2.25Derive the simplest sum-of-products expression for the function

$f = x_1' x_3' x_5' + x_1' x_3' x_4' + x_1' x_4 x_5 + x_1 x_2' x_3' x_5$

$= x_1' (x_3' x_5' + x_3' x_4' + x_4 x_5) + x_1 x_2' x_3' x_5$

$= x_1' ((x_3' x_5' + x_4 x_5) + x_3' x_4') + x_1 x_2' x_3' x_5$

$= x_1' ((x_3' x_5' + x_4 x_5) + x_3' x_4') + x_1 x_2' x_3' x_5$

Letting $x = x_5'$, $y = x_3'$, and $z = x_4$, we get

$f = x_1' ((x y + x' z) + x_3' x_4') + x_1 x_2' x_3' x_5$

$= x_1' ((x y + y z + x' z) + x_3' x_4') + x_1 x_2' x_3' x_5$ **obtained by using the consensus property**

$= x_1' ((x_5' x_3' + x_3' x_4 + x_5 x_4) + x_3' x_4') + x_1 x_2' x_3' x_5$

$= x_1' ((x_3' x_5' + x_3' x_4 + x_4 x_5) + x_3' x_4') + x_1 x_2' x_3' x_5$

$= x_1' (x_3' x_5' + x_3' x_4 + x_4 x_5 + x_3' x_4') + x_1 x_2' x_3' x_5$

$= x_1' (x_3' (x_5' + x_4 + x_4') + x_4 x_5) + x_1 x_2' x_3' x_5$

$= x_1' x_3' + x_1' x_4 x_5 + x_1 x_2' x_3' x_5$

$= x_1' x_4 x_5 + x_3' (x_1' + x_1 x_2' x_5)$

$= x_1' x_4 x_5 + x_3' (x_1' + x_2' x_5)$

$= x_1' x_3' + x_1' x_4 x_5 + x_2' x_3' x_5$

2.28 Design the simplest circuit that has three inputs, x1, x2, and x3, which produces an output value of 1 whenever two or more of the input variables have the value 1; otherwise, the output has to be 0.

Solution:

Truth table:

x_1	x_2	x_3	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1

1	1	0	1
1	1	1	1

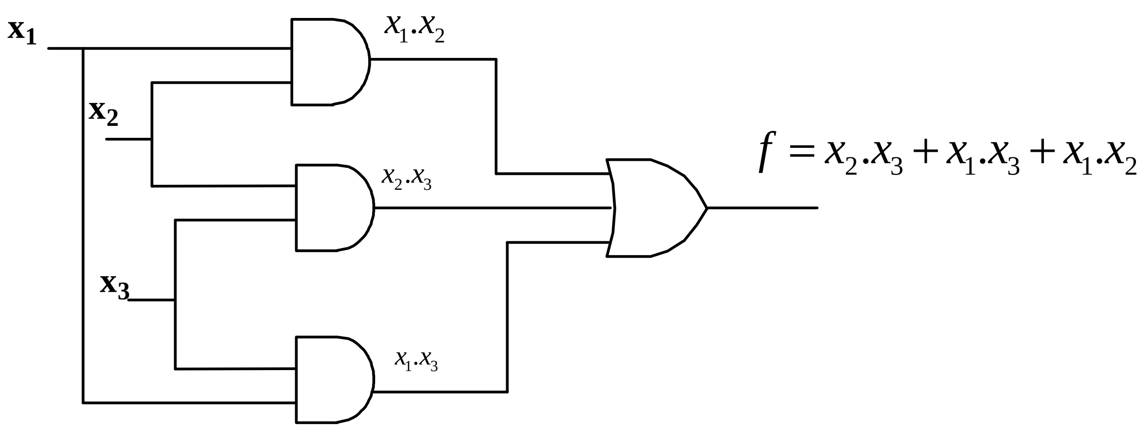
$f(x_1, x_2, x_3) = \sum m(3,5,6,7) = m_3 + m_5 + m_6 + m_7$

$\Rightarrow f = \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.x_3 + x_1.x_2.\overline{x_3} + x_1.x_2.x_3$

$\Rightarrow f = \overline{x_1}.x_2.x_3 + x_1.x_2.x_3 + x_1.\overline{x_2}.x_3 + x_1.x_2.x_3 + x_1.x_2.\overline{x_3} + x_1.x_2.x_3$

$\Rightarrow f = x_2.x_3(\overline{x_1} + x_1) + x_1.x_3(\overline{x_2} + x_2) + x_1.x_2(x_3 + \overline{x_3})$

$\Rightarrow f = x_2.x_3 + x_1.x_3 + x_1.x_2$



2.32 For the timing diagram in Figure P2.3, synthesize the function $f(x_1, x_2, x_3)$ in the simplest product-of-sums form.

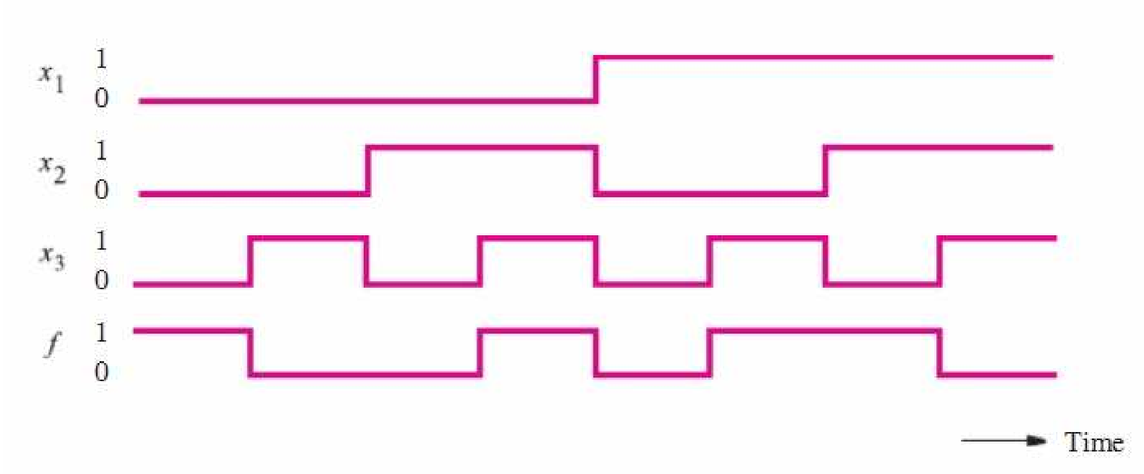


Figure P2.3 A timing diagram representing a logic function.

x_1	x_2	x_3	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1

1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$$f(x_1, x_2, x_3) = \prod M(1,2,4,7) = M_1.M_2.M_4.M_7$$

$$f = (x_1 + x_2 + \overline{x_3})(x_1 + \overline{x_2} + x_3)(\overline{x_1} + x_2 + x_3)(\overline{x_1} + \overline{x_2} + \overline{x_3})$$

2.33 For the timing diagram in Figure P2.4, synthesize the function $f(x_1, x_2, x_3)$ in the simplest sum-of-products form.

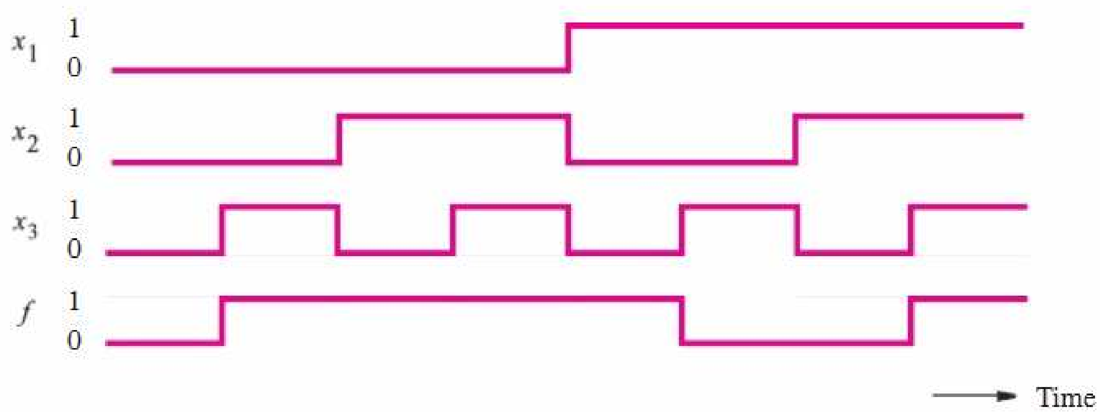


Figure P2.4 A timing diagram representing a logic function.

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$$f(x_1, x_2, x_3) = \sum m(1,2,3,4,7) = m_1 + m_2 + m_3 + m_4 + m_7$$

$$\Rightarrow f = \overline{x_1}.\overline{x_2}.x_3 + \overline{x_1}.x_2.\overline{x_3} + \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.\overline{x_3} + x_1.x_2.x_3$$

$$\Rightarrow f = \overline{x_1}.\overline{x_2}.x_3 + \overline{x_1}.x_2.x_3 + \overline{x_1}.x_2.\overline{x_3} + \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.\overline{x_3} + x_1.x_2.x_3 + \overline{x_1}.x_2.x_3$$

$$\Rightarrow f = \overline{x_1}.x_3(\overline{x_2} + x_2) + \overline{x_1}.x_2.(\overline{x_3} + x_3) + x_1.\overline{x_2}.\overline{x_3} + x_2.x_3(x_1 + \overline{x_1})$$

$$\Rightarrow f = \overline{x_1}.x_3 + \overline{x_1}.x_2 + x_1.\overline{x_2}.\overline{x_3} + x_2.x_3$$

2.40 Design the simplest circuit that implements the function $f(x_1, x_2, x_3) = \sum m(3,4,6,7)$ using NAND gates.

Solution:

$$f(x_1, x_2, x_3) = \sum m(3,4,6,7) = m_3 + m_4 + m_6 + m_7$$

$$\Rightarrow f = \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.\overline{x_3} + x_1.x_2.\overline{x_3} + x_1.x_2.x_3$$

$$\Rightarrow f = \overline{x_1}.x_2.x_3 + x_1.x_2.x_3 + x_1.\overline{x_2}.\overline{x_3} + x_1.x_2.\overline{x_3}$$

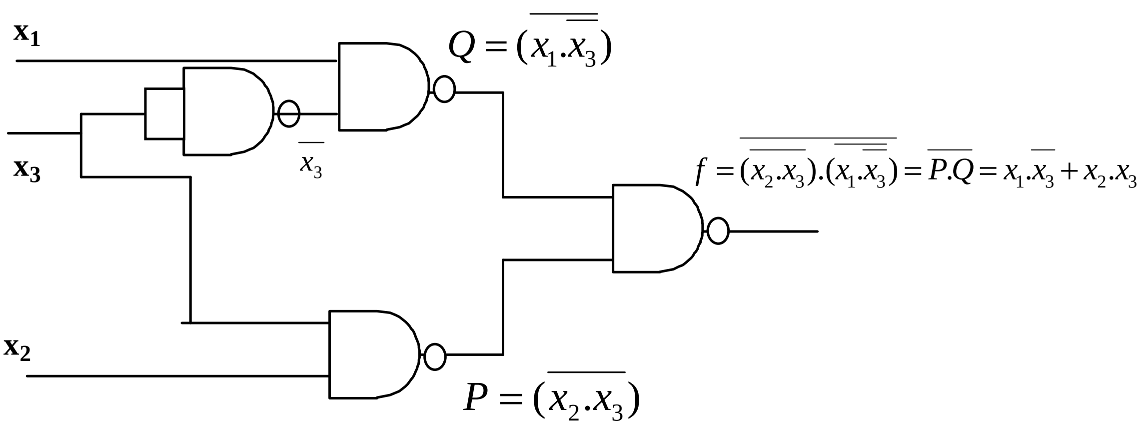
$$\Rightarrow f = x_2.x_3(\overline{x_1} + x_1) + x_1.\overline{x_3}(\overline{x_2} + x_2) = x_2.x_3 + x_1.\overline{x_3}$$

Design Using NAND gates:

$$\overline{f} = \overline{x_2.x_3 + x_1.\overline{x_3}} = (\overline{x_2.x_3}).(\overline{x_1.\overline{x_3}})$$

$$\therefore f = \overline{(\overline{x_2.x_3}).(\overline{x_1.\overline{x_3}})} = \overline{P.Q}$$

With $P = (\overline{x_2.x_3})$ and $Q = (\overline{x_1.\overline{x_3}})$



2.45 Use algebraic manipulation to derive the minimum sum-of-products expression for the function $f = \overline{x_1}.\overline{x_2}.x_3 + x_1.x_3 + x_2.x_3 + x_1.x_2.\overline{x_3}$

Solution:

$$f = \overline{x_1}.\overline{x_2}.x_3 + x_1.x_3 + x_2.x_3 + x_1.x_2.\overline{x_3}$$

$$\Rightarrow f = x_3(\overline{x_1}.\overline{x_2} + x_1) + x_2(x_3 + x_1.\overline{x_3})$$

$$\Rightarrow f = x_3(\overline{x_2} + x_1) + x_2(x_3 + x_1)$$

$$\Rightarrow f = x_3.\overline{x_2} + x_3.x_1 + x_2.x_3 + x_1x_2$$

$$\Rightarrow f = x_3(\overline{x_2} + x_1 + x_2) + x_1x_2$$

$$\Rightarrow f = x_3(1 + x_1) + x_1x_2$$

$$\Rightarrow f = x_3(1) + x_1x_2$$

$$\Rightarrow f = x_3 + x_1x_2$$