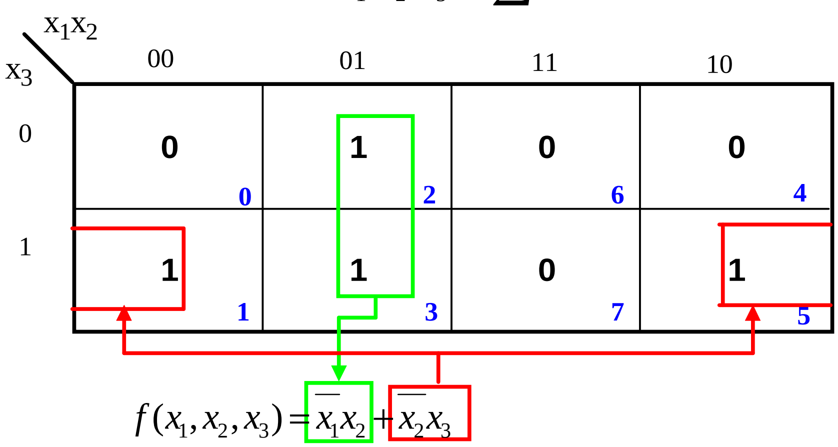
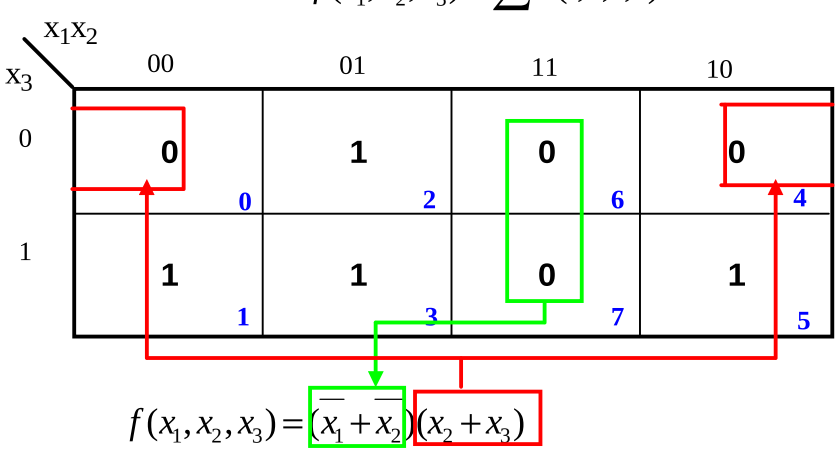
4.1 Find the minimum-cost SOP and POS forms for the function $f(x_1, x_2, x_3) = \sum m(1, 2, 3, 5)$.
Solution:

$$f(x_1, x_2, x_3) = \sum m(1,2,3,5)$$



Mapping the 1's from the K-map as follows
$f(x_1, x_2, x_3) = \overline{x}_1 x_2 + \overline{x}_2 x_3$ - This is the minimum-cost SOP form.

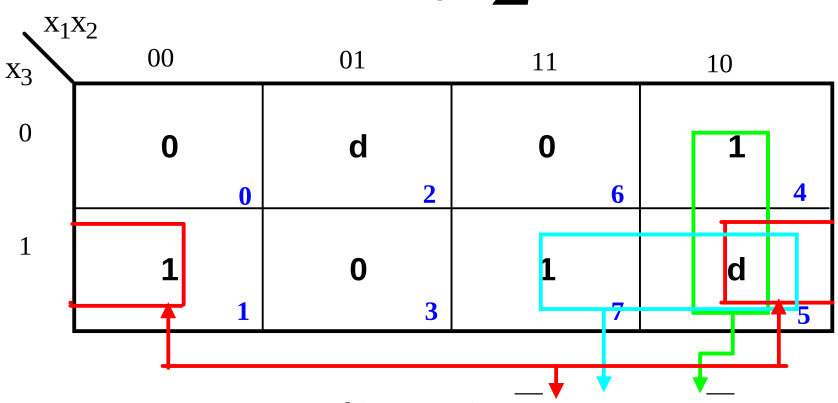
$$f(x_1, x_2, x_3) = \sum m(1,2,3,5)$$



Mapping the 0's from the K-map as follows
$f(x_1, x_2, x_3) = (\overline{x}_1 + \overline{x}_2)(x_2 + x_3)$ - This is the minimum-cost POS form.

4.2 Repeat problem 4.1 for the function $f(x_1, x_2, x_3) = \sum m(1, 4, 7) + D(2, 5)$.
Solution:

$$f(x_1, x_2, x_3) = \sum m(1,4,7) + D(2,5)$$

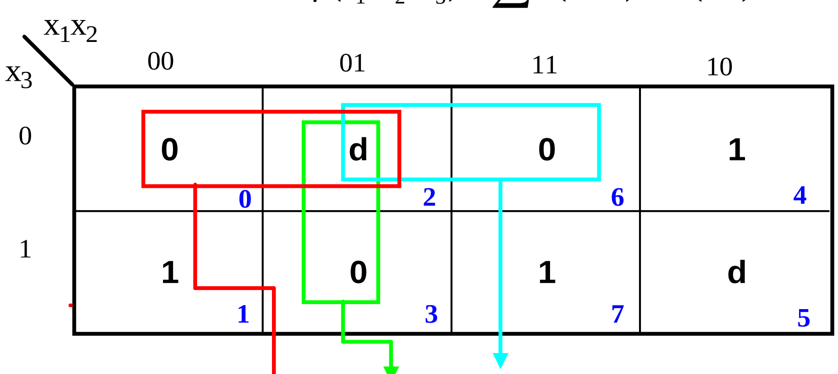


$$f(x_1, x_2, x_3) = \overline{x_2} x_3 + x_1 x_3 + x_1 \overline{x_2}$$

Mapping the 1's from the K-map as follows

$f(x_1, x_2, x_3) = \overline{x_2} x_3 + x_1 x_3 + x_1 \overline{x_2}$ -This is the minimum-cost SOP form.

$$f(x_1, x_2, x_3) = \sum m(1,4,7) + D(2,5)$$



$$f(x_1, x_2, x_3) = (x_1 + x_3)(x_1 + \overline{x_2})(\overline{x_2} + x_3)$$

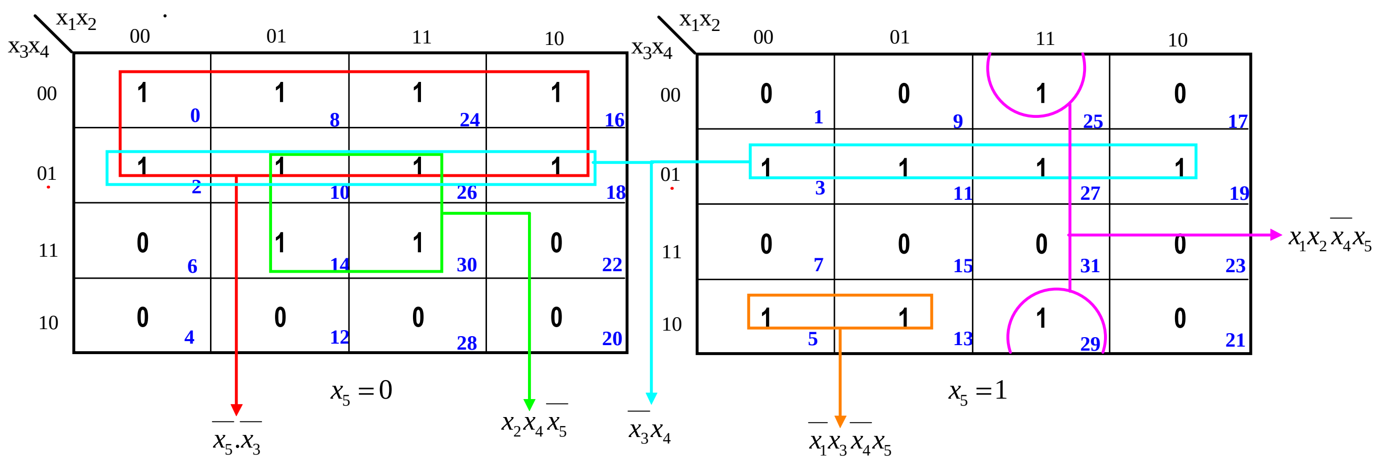
Mapping the 0's from the K-map as follows

$f(x_1, x_2, x_3) = (\overline{x_2} + x_3)(x_1 + x_3)(x_1 + \overline{x_2})$ This is the minimum-cost POS form.

4.5 Repeat problem 4.1 for the function $f(x_1, \ldots, x_5) = \prod M(1, 4, 6, 7, 9, 12, 15, 17, 20, 21, 22, 23, 28, 31)$.

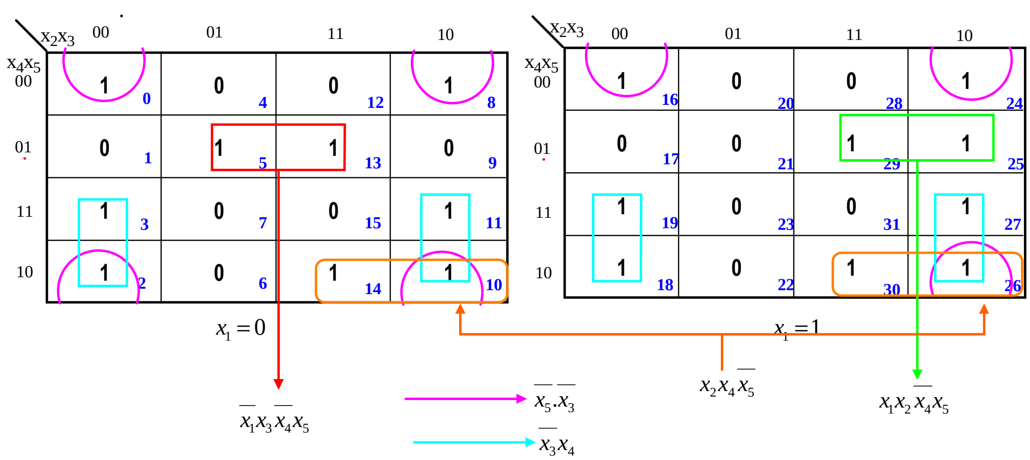
Solution:

$$f(x_1,x_2,x_3,x_4,x_5)=\prod M(1,4,6,7,9,12,15,17,20,21,22,23,28,31)$$



or

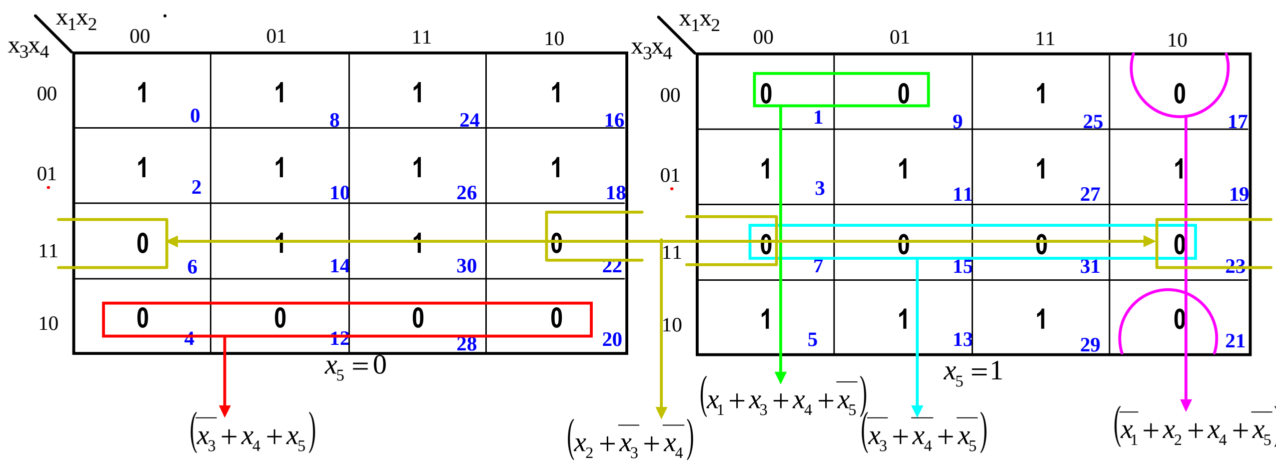
$$f(x_1,x_2,x_3,x_4,x_5)=\prod M(1,4,6,7,9,12,15,17,20,21,22,23,28,31)$$



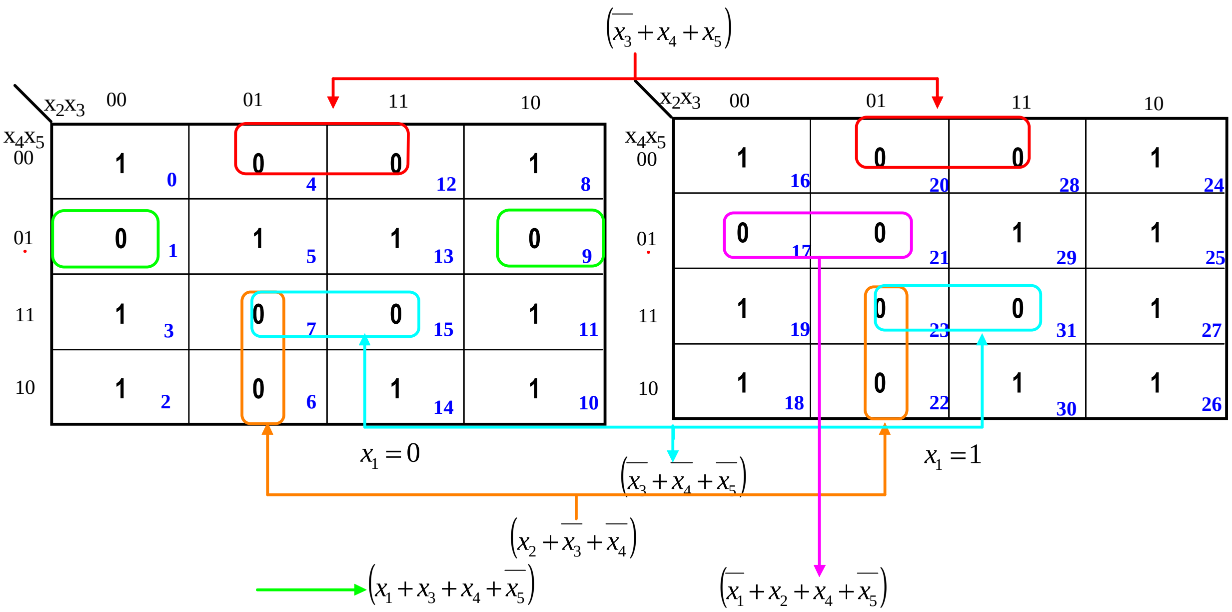
Mapping the 1's from the K-map as follows

$f(x_1,x_2,x_3,x_4,x_5)=\bar{x}_3.\bar{x}_5+x_2x_4\bar{x}_5+\bar{x}_3x_4+\bar{x}_1x_3\bar{x}_4x_5+x_1x_2\bar{x}_4x_5$ This is the minimum-cost SOP form.

$$f(x_1,x_2,x_3,x_4,x_5) = \prod M(1,4,6,7,9,12,15,17,20,21,22,23,28,31)$$



or



Mapping the 0's from the K-map as follows

$$f(x_1,x_2,x_3,x_4,x_5) = (\overline{x}_3 + x_4 + x_5)(x_2 + \overline{x}_3 + \overline{x}_4)(\overline{x}_3 + \overline{x}_4 + \overline{x}_5)(x_1 + x_3 + x_4 + \overline{x}_5)(\overline{x}_1 + x_2 + x_4 + \overline{x}_5)$$

This is the minimum-cost POS form

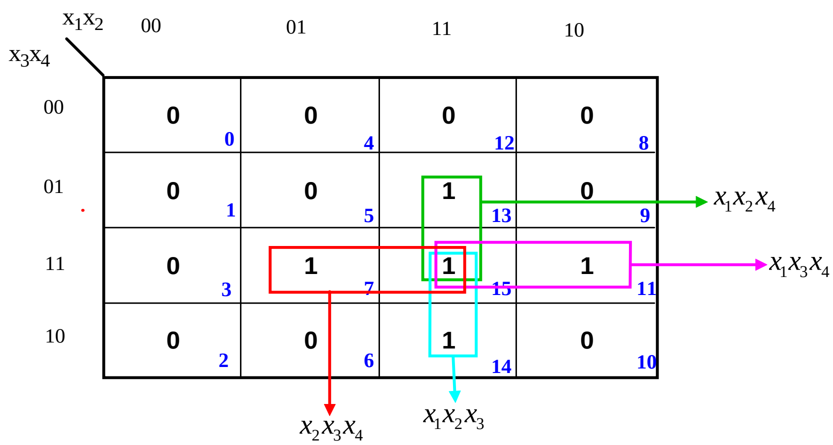
4.9 A four-variable logic function that is equal to 1 if any three or all four of its variables are equal to 1 is called a *majority* function. Design a minimum-cost SOPcircuit that implements this majority function.

Solution:

Truth table: $f(x_1,x_2,x_3,x_4) = \sum m(7,11,13,14,15)$

x1	x2	x3	x4	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$$f(x_1,x_2,x_3,x_4)=\sum m(7,11,13,14,15)$$



Mapping the 1's from the K-map as follows

$f(x_1,x_2,x_3) = x_1x_2x_3 + x_1x_2x_4 + x_2x_3x_4 + x_1x_3x_4$ -This is the minimum-cost SOP form.

4.12 A circuit with two outputs has to implement the following functions

f (*x*1, . . . , *x*4) =∑*m*(0, 2, 4, 6, 7, 9) + *D*(10, 11)

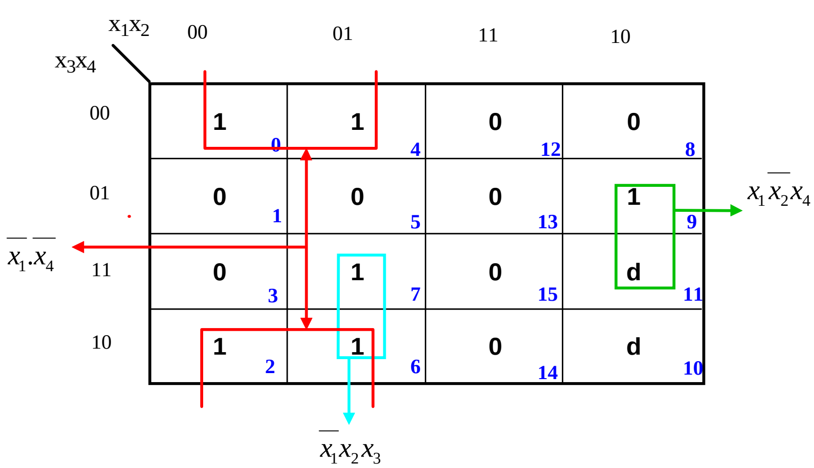
g(*x*1, . . . , *x*4) =∑*m*(2, 4, 9, 10, 15) + *D*(0, 13, 14)

Design the minimum-cost circuit and compare its cost with combined costs of two circuits that implement *f* and *g* separately. Assume that the input variables are available in both uncomplemented and complemented forms.

Solution:

The Karnaugh Map for getting the minimum cost SOP form for the function f is:

$$f(x_1,x_2,x_3,x_4)=\sum m(0,2,4,6,7,9)+D(10,11)$$



From the Karnaugh map the minimum cost SOP expression is

$f(x_1,x_2,x_3,x_4) = \bar{x}_1.\bar{x}_4 + \bar{x}_1x_2x_3 + x_1\bar{x}_2x_4$

The realization of function requires,

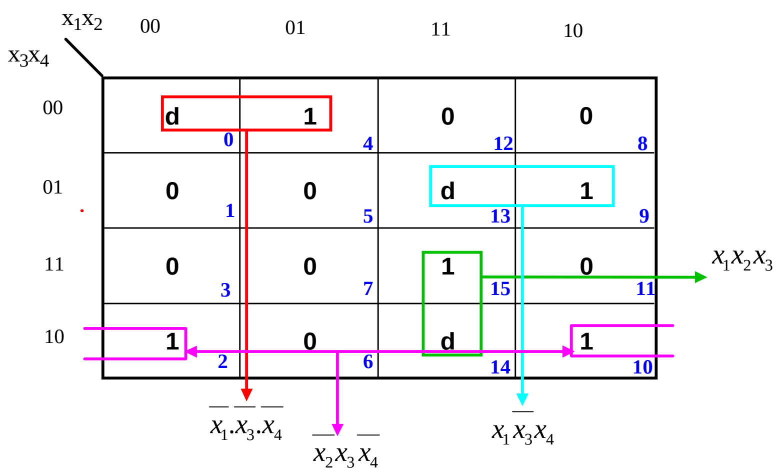
Two 3-input AND gate.

One 2-input AND gate.

One 3-input OR gate.

The number of gates needed to implementation the function is:
The cost of f=total no. of gates+ total no of inputs=(2+1+1)+{(2)(3)+(2)(1)+(3)(1)}=15

$$g(x_1, x_2, x_3, x_4) = \sum m(2,4,9,10,15) + D(0,13,14)$$



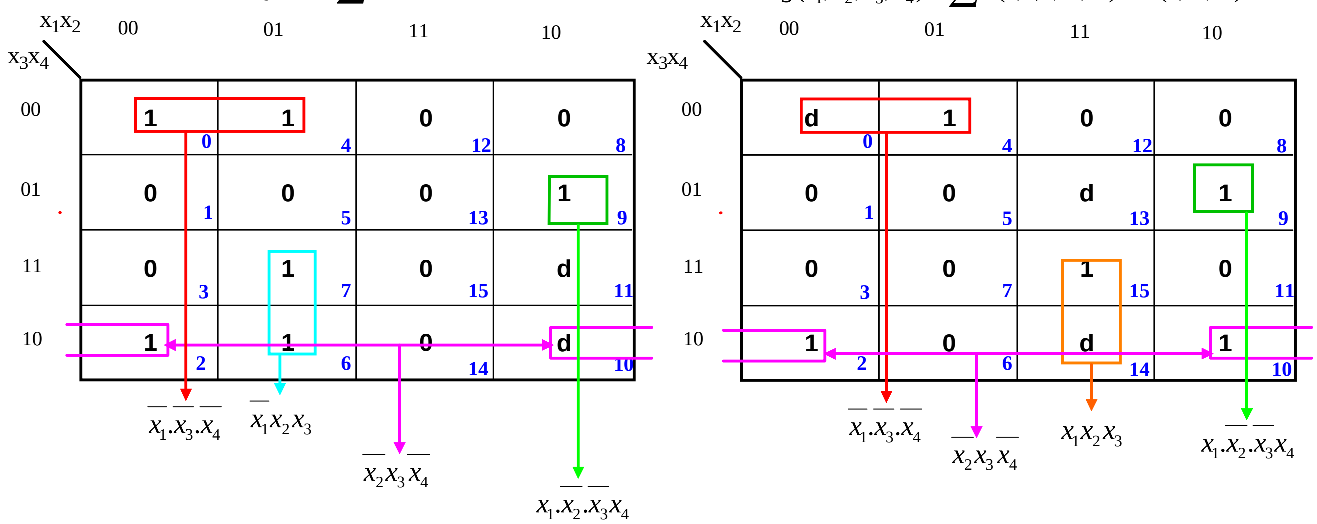
From the Karnaugh map the minimum cost SOP expression is

$$g(x_1, x_2, x_3, x_4) = \bar{x}_1.\bar{x}_3.\bar{x}_4 + \bar{x}_2 x_3 \bar{x}_4 + x_1 \bar{x}_3 x_4 + x_1 x_2 x_3$$

The realization of this function requires,
Four 3-input AND gate.
One 4-input OR gate.
The number of gates needed to implementation the function is:
The cost of g=total no. of gates+ total no of inputs=(4+1)+{(3)(4)+(4)(1)}=21

The total cost of realization of function f and g is=15+21=36
In order to find the combined cost of implementing both the function f and g, we draw the Karnaugh map of f and g and groped the elements such that both the function have as many common groping as they possibly can have. Hence the resultant modified Karnaugh maps

$f(x_1,x_2,x_3,x_4)=\sum m(0,2,4,6,7,9)+D(10,11)$ $\quad$ $g(x_1,x_2,x_3,x_4)=\sum m(2,4,9,10,15)+D(0,13,14)$



Modified minimum cost SOP forms are

$$f(x_1,x_2,x_3,x_4)=\boxed{\bar{x}_1\cdot\bar{x}_3\cdot\bar{x}_4+\bar{x}_2x_3\bar{x}_4+x_1\bar{x}_2\cdot\bar{x}_3x_4}+\bar{x}_1x_2x_3$$
$$g(x_1,x_2,x_3,x_4)=\boxed{\bar{x}_1\cdot\bar{x}_3\cdot\bar{x}_4+\bar{x}_2x_3\bar{x}_4+x_1\bar{x}_2\cdot\bar{x}_3x_4}+x_1x_2x_3$$

Hence the first three terms are shared between the two functions.
The implementation of both the functions require:
Four 3-input AND gate
One 4-input AND gate
Two 4-input OR gate
Now the combine cost of the system is
The cost of system=total no. of gates+ total no of inputs=(4+1+2)+{(3)(4)+(4)(1)+(4)(2)}=31

4.14 Implement the logic circuit in Figure 4.23 using NAND gates only.

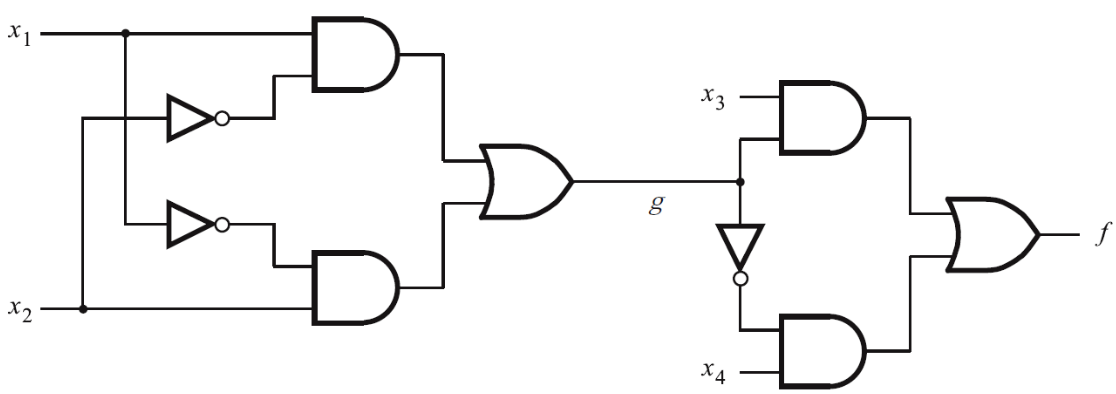


Figure 4.23 Logic circuit for Example 4.6.

Solution: the followings are the logic functions for g and f:

$$g(x_1, x_2) = x_1.\overline{x_2} + \overline{x_1}x_2$$

$$f(x_3, x_4, g) = x_3 g + \overline{g}x_4$$

Now implementing the logic using only NAND gate can be done by considering the following relationships

$$\overline{A} = (\overline{A.A}) = (A \uparrow A)$$

$$A + B = \overline{\overline{A}.\overline{B}} = (A \uparrow A) \uparrow (B \uparrow B)$$

$$AB = \overline{\overline{AB}} = (A \uparrow B) \uparrow (A \uparrow B)$$

Implementation of given logic function using only NAND gate is:

$$g(x_1, x_2) = x_1.\overline{x_2} + \overline{x_1}x_2 = \overline{\overline{x_1.\overline{x_2} + \overline{x_1}x_2}} = \overline{\left(\overline{x_1.\overline{x_2}}\right)\left(\overline{\overline{x_1}x_2}\right)}$$

$$\overline{x_1.\overline{x_2}} = \overline{x_1.\overline{x_2.x_2}} = x_1 \uparrow (x_2 \uparrow x_2)$$

$$\overline{\overline{x_1}x_2} = \overline{\overline{x_1.x_1}.x_2} = (x_1 \uparrow x_1) \uparrow x_2$$

$$g = [(x_1 \uparrow (x_2 \uparrow x_2))] \uparrow [(x_1 \uparrow x_1) \uparrow x_2]$$

$$f(x_3, x_4, g) = x_3 g + \overline{g}x_4 = \overline{\left(\overline{x_3 g + \overline{g}x_4}\right)} = \overline{\overline{(x_3 g)}.\overline{(\overline{g}x_4)}}$$

$$\overline{(x_3 g)} = x_3 \uparrow g$$

$$\overline{(\overline{g}x_4)} = \overline{g} \uparrow x_4 = (g \uparrow g) \uparrow x_4$$

$$f = [x_3 \uparrow g] \uparrow [(g \uparrow g) \uparrow x_4]$$