1. Use Quine-McCluskey Procedure to find all prime implicants for the functions given below:

a. f(w,x,y,z) = ∑ (1,3,6,9,11,14) + dc = ∑ (7,12,13,15)

Solution:

1- Cell		2-Cell		4-Cell	
1(0001)	√	1,3(2)	√	1,3,9,11(2,8)	**A**
		1,9(8)	√		
3(0011)	√	3,7(4)	√	3,7,11,15(4,8)	**B**
6(0110)	√	3,11(8)	√	6,7,14,15(1,8)	**C**
9(1001)	√	6,7(1)	√	9,11,13,15(2,4)	**D**
12(1100)	√	6,14(8)	√	12,13,14,15(1,2)	**E**
		9,11(2)	√		
		9,13(4)	√		
		12,13(1)	√		
		12,14(2)	√		
7(0111)	√	7,15(8)	√		
11(1011)	√	11,15(4)	√		
13(1110)	√	13,15(2)	√		
14(1110)	√	14,15(1)	√		
15(1111)	√				

Prime Implicants	wxyz	
A→1,3,9,11(2,8)	_0_1	$\bar{x}z$
B→3,7,11,15(4,8)	_ _11	yz
C→6,7,14,15(1,8)	_11_	xy
D→9,11,13,15(2,4)	1_ _1	wz
E→12,13,14,15(1,2)	11_ _	wx

b. f(w,x,y,z) = $\prod$(1,3,5,7,12,13,15) + dc = $\sum$ (6,14)

Solution:

1- Cell		2-Cell		4-Cell	
1(0001)	√	1,3(2)	√	1,3,5,7(2,4)	**A**
		1,5(4)	√		
3(0011)	√	3,7(4)	√	5,7,13,15(2,8)	**B**
5(0101)	√	5,7(2)	√	6,7,14,15(1,8)	**C**
6(0110)	√	5,13(8)	√	12,13,14,15(1,2)	**D**
12(1100)	√	6,7(1)	√		
		6,14(8)	√		
		12,13(1)	√		
		12,14(2)	√		
7(0111)	√	7,15(8)	√		
13(1110)	√	13,15(2)	√		
14(1110)	√	14,15(1)	√		
15(1111)	√				

Prime Implicants	wxyz	
A→1,3,5,7(2,4)	0_ _1	$w+\bar{z}$
B→5,7,13,15(2,4)	_ 1_1	$\bar{x}+\bar{z}$
C→6,7,14,15(1,8)	_11_	$\bar{x}+\bar{y}$
D→12,13,14,15(1,2)	11_ _	$\bar{w}+\bar{x}$

2. Reduce the following prime table and develop the complete Petrick Function

F1

	C_1	C_2	C_3	C_4	C_5	Cost
A	X					2
B	X			X		4
C		X		X	X	5
D	X	X		X		3
E			X		X	3
F		X	X			4

Solution:

F1

	C_1	C_2	C_3	C_4	C_5	Cost
A	X					2
B	X			X		4
C		X		X	X	5
D	X	X		X		3
E			X		X	3
F		X	X			4

(1) No primary essentials.

(2) No column elimination.

(3) Row D dominates row B, delete row B.

	C_1	C_2	C_3	C_4	C_5	Cost
A	X					2
B	X			X		4
C		X		X	X	5
D	X	X		X		3
E			X		X	3
F		X	X			4

⇨

	C_1	C_2	C_3	C_4	C_5	Cost
A	X					2
C		X		X	X	5
D	X	X		X		3
E			X		X	3
F		X	X			4

(4) Column 2 dominates column 4, delete column 2.

	C_1	C_2	C_3	C_4	C_5	Cost
A	X					2
C		X		X	X	5
D	X	X		X		3
E			X		X	3
F		X	X			4

⇨

	C_1	C_3	C_4	C_5	Cost
A	X				2
C			X	X	5
D	X		X		3
E		X		X	3
F		X			4

(5) Row E dominates row F and is cheaper, delete row F.

	C_1	C_3	C_4	C_5	Cost
A	X				2
C			X	X	5
D	X		X		3
E		X		X	3
F		X			4

⇨

	C_1	C_3	C_4	C_5	Cost
A	X				2
C			X	X	5
D	X		X		3
E		X		X	3

(6) Column 5 dominates column 3, delete column 5.

	C_1	C_3	C_4	C_5	Cost
A	X				2
C			X	X	5
D	X		X		3
E		X		X	3

⇨

	C_1	C_3	C_4	Cost
A	X			2
C			X	5
D	X		X	3
E		X		3

(7) Column C3 now has one X, E is essential.

(8) Row D dominates row C and it is cheaper, delete row C(higher cost than A).

	C_1	C_4	Cost
A	X		2
C		X	5
D	X	X	3

⇨

	C_1	C_4	Cost
A	X		2
D	X	X	3

(9) Column 1 dominates column 4, delete column 1. D is essential.

	C_1	C_4	Cost
A	X		2
D	X	X	3

⇨

	C_4	Cost
A		2
D	X	3

P=D.E
Cost=3+3=6

In Petrick's method, a *Boolean expression* P is formed which describes all possible solutions of the table.

$$P = (A+B+D)(C+D+F)(E+F)(B+C+D)(C+E)$$

$$\Rightarrow P = (A+B+D)(C+D+F)(B+C+D)(E+F)(C+E)$$

$$\Rightarrow P = (A+B+D)(C+D+BF)(E+CF)$$

$$\Rightarrow P = [D+(A+B)(C+BF)](E+CF)$$

$$\Rightarrow P = (D+AC+BC+ABF+BF)(E+CF)$$

$$\Rightarrow P = DE+ACE+BCE+ABEF+BEF+CDF+ACF+BCF+ABCF$$

Here the products with fewer term is DE
Minimal Cover={D,E} with cost=3+3=6

3. Perform a tagged Quine-McCluskey reduction to obtain the prime implicants for the following multi-functions.

$f1(w,x,y,z) = \sum(0,1,2,6,7,13) \quad + \text{d.c.} = \sum(3,11,15)$

$f2(w,x,y,z) = \sum(4,8,12,15) \quad + \text{d.c.} = \sum(0,1,7)$

Solution:

1- Cell		2-Cell		4-Cell	
0(0000)	f1f2√	0,1(1)f1f2 0,2(2)f1 _ 0,4(4)_ f2 0,8(8)_ f2	E √ √	0,1,2,3(1,2)f1 _ 0,4,8,12(4,8)_ f2	**A** B
1(0001) 2(0010) 4(0100) 8(1000)	f1f2√ f1 _√ _ f2√ _ f2√	1,3(2)f1 _ 2,3(1)f1 _ 2,6(4)f1 _ 4,12(8)_ f2 8,12(4)_ f2	√ √ √ √ √	2,3,6,7(1,4)f1 _	**C**
3(0011) 6(0110) 12(1100)	f1 _√ f1 _√ _ f2√	3,7(4)f1 _ 3,11(8)f1 _ 6,7(1)f1 _	√ √ √	3,7,11,15(4,8)f1 _	**D**
7(0111) 11(1011) 13(1110)	f1f2√ f1 _√ f1 _√	7,15(8)f1f2 11,15(4)f1 _ 13,15(2)f1 _	F √ G		
15(1111)	f1f2√				

Prime Implicants	wxyz	
A→0,1,2,3(1,2)f1	00_ _	$\overline{w}\overline{x}$ →f1
B→0,4,8,12(4,8)f2	_ _ 00	$\overline{y}\overline{z}$ →f2
C→2,3,6,7(1,4)f1	0_1_	$\overline{w}y$ →f1
D→3,7,11,15(4,8)f1	_ _11	yz →f1
E→0,1(1)f1f2	000_	$\overline{w}\overline{x}\overline{y}$ →f1f2
F→7,15(8)f1f2	011_	$\overline{w}xy$ →f1f2
G→13,15(2)f1	_111	xyz →f1

4. Perform prime implicant table reduction using Criterion 3 and find the Petrick function.

	F1				F2				
	C_1	C_2	C_3	C_4	C5	C6	C7	C8	Cost
A	X								2
B	X	X							3
C		X			X		X		3
D			X			X			2
E	X				X		X		3
F		X		X			X	X	3
G		X			X				3

Solution:

	F1				F2				
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Cost
A	X								2
B	X	X							3
C		X			X		X		3
* D			X			X			2
E	X				X		X		3
* F		X		X			X	X	3
G		X			X				3

⇨

	F1	F2	
	C_1	C_5	Cost
A	X		2
B	X		3
C		X	3
E	X	X	3
G		X	3

1. C_3, C_4,C_6 and C_8 have lone 'X', these are essentials , delete columns. Delete the row D and F, since it has only the lone 'X's. Reduce the cost to 1. Delete columns C_2 and C_7.
2. Row E dominates row A,B,C,G (in F1 and F2) and has the same, delete A,B,C,G. Reduce the cost to 1.

	F1	F2	
	C_1	C_5	Cost
A	~~X~~		2
B	~~X~~		3
C		~~X~~	3
E	X	X	1
G		~~X~~	3

$$P = D_1F_1D_2F_2E_1E_2$$

Cost=2+3+1+1+3+1=11

	F_1				F_2				
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Cost
A	X								2
B	X	X							3
C		X			X		X		3
D			X			X			~~2~~ 1
E	X				X		X		~~3~~ 1
F		X		X			X	X	~~3~~ 1
G		X			X				3

5. Perform prime implicant table reduction using (only) the rules for essential primes for Criterion 3.

	F1					F2				
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	Cost
A	X		X							3
B	X	X								2
C						X	X			2
D						X				4
E										2
F	X	X		X		X		X		2
G		X			X				X	2
H	X	X	X				X			2

Solution:

	F1					F2				
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	cost
A	X		X							3
B	X	X								2
C						X	X			2
D						X				4
E										2
F	X	X		X		X		X		2
G		X			X				X	2
H	X	X	X				X			2

⇒

	F1	F2		
	C_3	C_6	C_7	cost
A	X			3
B				2
C		X	X	2
D		X		4
E				
H	X		X	2

1. C_4, C_5, C_8, C_9 has lone 'X', these are essentials, delete columns. Delete the row G and F, since it has only the lone 'X's. Reduce the cost to 1. Delete columns C_1,C_2 and C_6.
2. Row H dominates A, delete A.
3. Row C dominates D, delete D.

	F1	F2		
	C_3	C_6	C_7	cost
A	X			3
C		X	X	2
D		X		4
H	X		X	2

⇒

	F1	F2		
	C_3	C_6	C_7	cost
C		X	X	2
H	X		X	2

	F1	F2	
	C_3	C_6	cost
C		X	2
H	X		2

4. Column C_7 dominates C_6. Delete C_7.

$$P = G_1F_1G_2F_2(H_1 + C_2)$$

Cost=2+1+2+1+2+1=9